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1/1983

INSTYTUT PODSTAWOWYCH PROBLEMÓW TECHNIKI
POLSKIEJ AKADEMII NAUK

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WARSZAWA 1983

ISSN 0208-5658

Praca wpłynęła do Redakcji dnia 11 października 1982 r.

Na prawach rękopisu

Instytut Podstawowych Problemów Techniki PAN
Nakład 150 egz. Ark.wyd. 0,8. Ark.druk.1,25.
Oddano do drukarni w styczniu 1983 r.
Nr zamówienia 146/o/83

Warszawska Drukarnia Naukowa, Warszawa,
ul.Śniadeckich 8

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THE INFLUENCE OF SOME SECOND ORDER EFFECTS ON THE BEHAVIOUR OF RIGID PLASTIC SHELLS AT THE YIELD POINT LOAD

1. Introduction.

Whereas it is rather obvious that the problem of stability and post yield behaviour of a structure can be treated properly only within framework of the geometrically non-linear theory, the question arises, whether the second order effects, usually neglected when formulating the constitutive relations, play also substantial role.

Therefore, such effects as influence of various definitions of perfectly plastic material /associated with the different objective stress rates/ on the post yield behaviour of the shells is considered in the paper. The material incompressibility, observed when plastic deformations take place, is also taken into account when formulating geometrical relations. The results are compared with the known solutions obtained under the assumption of constant shell thickness.

The problem considered is illustrated by examples of cylindrical shells subject to the end axial forces and uniformly distributed lateral dead load.

2. Material stability and the definition of perfectly plastic material.

In the recent literature the following two approaches to the definition of the material stability of time-independent materials are most commonly used.

The first one is concerned with Drucker's concept [1] . The material stability condition is then derived from the energy criterion of stability of a body or a system when homogeneous stress and strain states are assumed.

The other approach consists of generalization of the definition of material stability under one-dimensional tension to cover an arbitrary stress state. According to this definition the material is said to be stable when the stress-strain curve is rising and is said unstable when the curve is falling. This idea can be generalized for the 3-dimensional stress state and expressed analytically as a requirement that the scalar product of conjugate stress rate and strain rate tensors be non-negative for stable material

$$\overset{\triangledown}{\sigma}{}^{ij} d_{ij} \geqslant 0 \qquad /2.1/$$

where $\overset{\triangledown}{\underset{\sim}{\sigma}}$ denotes an objective time derivative of the Cauchy /true/ stress tensor and d_{ij} is the symmetric part of velocity gradient.

This definition does not, however, describe the material properties in a unique way since the material stability depends here upon the choice of the objective stress rate measure.

In the literature, the Jaumann stress derivative /denoted here by $\overset{\triangledown}{\underset{\sim}{\sigma}}{}^{J}$ / is usually chosen to be used in the constitutive relations. The Jaumann stress rate is associated with axes rotating with a material element but not deforming with it. The relation /2.1/ can then be rewritten in the form

$$\overset{\triangledown J}{\sigma}{}^{ij} d_{ij} \geqslant 0 \qquad /2.2/$$

and is called material stability condition in the Jaumann sense [2]. Using, however, the Oldroyd stress derivative $\overset{\triangledown o}{\underset{\sim}{\sigma}}$ /associated with the axes rotating and deforming with the material element/ the relation /2.1/ takes the form

$$\overset{\triangledown o}{\sigma}{}^{ij} d_{ij} \geqslant 0 \qquad /2.3/$$

and is called material stability condition in the Oldroyd sense [2].

Making use of the known relation between the Jaumann and the Oldroyd stress derivatives

$$\overset{\nabla_J}{\sigma}{}^{ij} = \overset{\nabla_O}{\sigma}{}^{ij} + \sigma^{ik} d_k^j + \sigma^{kj} d_k^i \, , \qquad /2.4/$$

the material stability condition /2.2/ can be transformed to become

$$\overset{\nabla_O}{\sigma}{}^{ij} d_{ij} + 2\sigma^{ij} v^m_{\cdot ;i} v_{m;j} \geqslant 0 \, . \qquad /2.5/$$

The theory of finite deformation of plastic shells is usually formulated in the total Langrangian description. Then, the constitutive relations are required to be established using variables of this description, that is Kirchhoff stress tensor S^{KL} and the Green strain tensor E_{KL} . It may be shown that the following relation takes place [2]

$$\overset{\nabla_O}{\sigma}{}^{ij} d_{ij} = \frac{\rho}{\rho_o} \dot{S}^{KL} \dot{E}_{KL} \qquad /2.6/$$

where ρ and ρ_o are the instantaneous and the initial mass densities.

Substituting /2.6/ into /2.3/ we obtain material stability condition in the Oldroyd sense in the form

$$\dot{S}^{KL} \dot{E}_{KL} \geqslant 0 \, . \qquad /2.7/$$

Similarly, in view of the relation

$$\overset{\nabla_J}{\sigma}{}^{ij} d_{ij} = \dot{S}^{KL} \dot{E}_{KL} + 2S^{KL} \dot{E}_{KM} \dot{E}^M_L \qquad /2.8/$$

the material stability condition in the Jaumann sense may be written as

$$\dot{S}^{KL} \dot{E}_{KL} + 2S^{KL} \dot{E}_{KM} \dot{E}^M_L \geqslant 0 \, . \qquad /2.9/$$

Since the perfectly plastic material is defined as a plastic solid for which the neutral material stability condition is satisfied, in view of above considerations, the following definitions of the perfectly plastic material can be introduced.

We say that material is perfectly plastic in the Jaumann sense if

$$\overset{\nabla\gamma}{\sigma}{}^{ij} d_{ij} = 0 \qquad /2.10/$$

or

$$\dot{S}^{KL}\dot{E}_{KL} + 2S^{KL}\dot{E}_{KM}\dot{E}^{M}_{L} = 0 . \qquad /2.11/$$

Similarly, we say that material is perfectly plastic in the Oldroyd sense if

$$\overset{\nabla o}{\sigma}{}^{ij} d_{ij} = 0 \qquad /2.12/$$

or

$$\dot{S}^{KL}\dot{E}_{KL} = 0 \qquad /2.13/$$

if the Lagrangian description is applied.

3. Geometrical stability.

A question of geometrical stability depends on the material stability, boundary conditions and the geometry of the body.

The geometrical instability problem arises when the instability associated with changes in geometry is great enough to overcome the stability of the material.

The criterion for geometrical stability may be formulated in terms of the dead load intensity rate [3] ; so let us consider a structure subject to a system of dead loads P of monotonically increasing intensity

$$\underset{\sim}{P}(\underset{\sim}{X},t) = \mu(t)\,\overset{\circ}{\underset{\sim}{P}}(\underset{\sim}{X}) . \qquad /3.1/$$

where $\mu(t)$ indicates the load intensity and $\overset{\circ}{\underset{\sim}{P}}(X)$ specifies the load distribution.

The rate of loading is therefore

$$\dot{\underset{\sim}{P}}(\underset{\sim}{X},t) = \dot{\mu}(t)\,\overset{\circ}{\underset{\sim}{P}}(\underset{\sim}{X}) . \qquad /3.2/$$

Geometrical stability is associated with $\dot{\mu} > 0$, and this means that a quasistatic motion of a structure takes place only for increasing dead loads; if $\dot{\mu} < 0$ the structure is said to be geometrically unstable and it means that a structure continues to deform plasticity also if decreasing dead loads are applied.

Making use of the Principle of Virtual Energy and the non-linear strain-displacement relation the following expression can be derived for the stress rates [4]

$$\int_S \dot{\mathring{P}}^K \dot{u}_K \, dS = \int_V (\dot{S}^{KL} \dot{E}_{KL} + S^{KL} \dot{u}_{M|K} \dot{u}^M_{\ |L}) \, dV \qquad /3.3/$$

where u is a displacement vector; vertical stroke denotes covariant differentiation; S and V are the surface and the volume of the body respectively in the original configuration.

Substituting /3.2/ into /3.3/, the rate of load intensity may be determined from the formula

$$\dot{\mu}(t) = \frac{\int_V (\dot{S}^{KL} \dot{E}_{KL} + S^{KL} \dot{u}_{M|K} \dot{u}^M_{\ |L}) \, dV}{\int_S \mathring{P}^K \dot{u}_K \, dS} . \qquad /3.4/$$

For positive $\mu(t)$ the following inequality is always satisfied

$$\int_S \mathring{P}^K \dot{u}_K \, dS \geqslant 0 . \qquad /3.5/$$

From /3.4/ and /3.5/ it follows that $\dot{\mu} > 0$ if

$$\int_V (\dot{S}^{KL} \dot{E}_{KL} + S^{KL} \dot{u}_{M|K} \dot{u}^M_{\ |L}) \, dV > 0 . \qquad /3.6/$$

Therefore, the relation /3.6/ can be considered as the geometrical stability condition.

For the rigid-perfectly plastic material in the Oldroyd sense, in view of /2.13/ the geometrical stability criterion /3.6/ reduces to

$$\int_V S^{KL} \dot{u}_{M|K} \dot{u}^M_{\ |L} \, dV > 0 , \qquad /3.7/$$

whereas for rigid-perfectly plastic material in the Jaumann sense, in view of /2.11/ it becomes

$$-\int_V S^{KL} \dot{u}_{K|M} \dot{u}^M_{\ |L} \, dV > 0 . \qquad /3.8/$$

4. Application of the geometrical stability conditions to cylindrical shells.

Let us consider a thin-walled, cylindrical shell subject to a rotationally symmetric deformation under action of uniformly distributed dead load P and the axial end load T, Fig. 1.

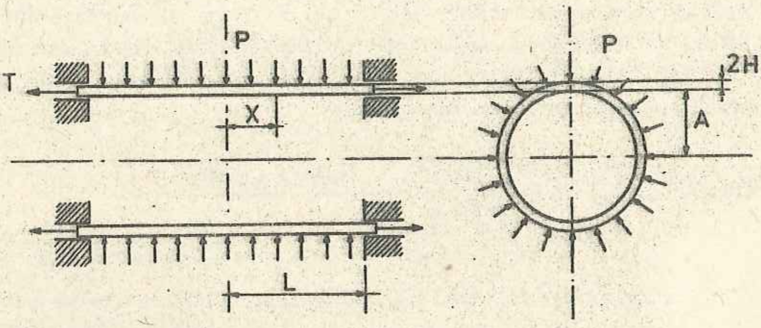

Fig. 1.

Assuming that the components of the displacement vector u^k are analytic functions of the coordinate Z normal to the shell middle surface, they can be expanded into a power series

$$
\begin{aligned}
u^z &= W(X) + Z\gamma^z(X) + \ldots\ldots \\
u^x &= V(X) + Z\gamma^x(X) + \ldots\ldots \qquad /4.1/ \\
u^\varphi &= U(X) + Z\gamma^\varphi(X) + \ldots\ldots
\end{aligned}
$$

In view of the assumption that the considered shells are thin, the non-linear terms with respect to Z can be neglected.

From the assumption of rotationally symmetric deformation it follows that the circumferential component u^φ of the displacement vector is equal zero.

Next, we assume that the transverse shearing is negligibly small and that no volume changes take place during plastic deformations. These assumptions allow us to express the functions γ^z

and T^x in terms of W and V.

Finally, the displacement field is find to has the form

$$
\begin{aligned}
u^z &= W - Z\left(V_{,x} + \frac{W}{A}\right) \\
u^x &= V - Z\, W_{,x} \\
u^\theta &= 0 .
\end{aligned}
\qquad /4.2/
$$

This means that the straight normals to the middle surface remain straight but the normality is satisfied only for $Z = 0$, moreover their length /thickness of shells/ can change during the deformation process.

The above kinematic assumptions are less restrictive than the assumptions of the classical Kirchhoff-Love theory of shells which say that the straight normals remain straight and normal and do not change their length. The displacement field has then the form:

$$
\begin{aligned}
u^z &= W \\
u^x &= V - Z\, W_{,x} \\
u^\theta &= 0 .
\end{aligned}
\qquad /4.3/
$$

Making use of /2.13/ and /4.2/ in /3.4/ the expression defining the rate of load intensity for perfectly plastic material in the Oldroyd sense may now be written as:

$$
\dot{\mu} = \frac{\int_S \left\{ n_\theta \dot{w}^2 + n_x \left[\left(\frac{A^2}{L^2}\right) \dot{w}_{,x}^2 + \dot{v}_{,x}^2 \right] - \frac{1}{\alpha}\left[m_x \left(\dot{w}_{,x}^2 + \dot{v}_{,xx} \dot{w}_{,x} + \dot{v}_{,x} \dot{w}_{,xx} \right) + m_{x,\dot{x}} \left(2\dot{w}_{,x} \dot{v}_{,x} + \dot{w}_{,x} \dot{w} \right) \right] \right\} dS}{\int_S \left(p\dot{w} + t\, \dot{v}_{|x=1} \right) dS} \qquad /4.4/
$$

where the dimentionless quantities are defined as follows

$$
w = \frac{W}{A}, \quad v = \frac{V}{L}, \quad x = \frac{X}{L}, \quad \alpha = \frac{L^2}{AH}, \qquad /4.5/
$$

$$n_x = \frac{1}{2\sigma_0 H}\int_{-H}^{+H} S_x^x \, dZ, \quad n_\theta = \frac{1}{2\sigma_0 H}\int_{-H}^{+H} S_\theta^\theta \, dZ, \quad m_x = \frac{1}{\sigma_0 H^2}\int_{-H}^{+H} S_x^x Z \, dZ \qquad /4.6/$$

$$p = \frac{PA}{2\sigma_0 H}, \quad t = \frac{T}{4\pi\sigma_0 AH}. \qquad /4.7/$$

Assuming the "limited interaction" yield condition, the limit load solution for the cylindrical shell with the boundary conditions considered in the paper was obtained by P.G. Hodge [5] .

The curve ABCDEF in Fig. 2 shows the yield-point loading curve for the particular case of $\alpha = \infty$ and the curve A'B'C'D'E'F' for $\alpha = 2$.

The yield-point loading curves for $2 < \alpha < \infty$ have the similar shape and are placed between the curves for $\alpha = 2$ and $\alpha = \infty$. In Fig. 2 are marked also the curves for $\alpha = 3$ and $\alpha = 5$.

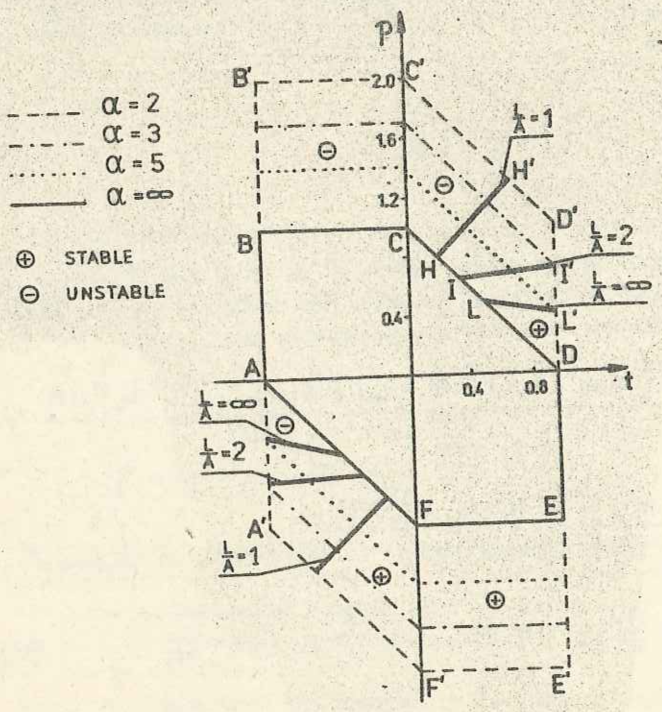

Fig. 2

Now, making use of stress and velocity fields given by limit load solution we shall supplement this solution by information about stability and the slope of the load-deflection curve at the yield-point load.

For the range of loads represented by the line CD or C'D' in Fig. 2, the limit load solution [5] gives:

$$
\begin{aligned}
&0 \leqslant t < 1 , \quad p = 1 - t + \frac{2}{\alpha} , \\
&m_x = 1 - 2x^2 , \quad n_\theta = -1 + t , \quad n_x = t , \\
&w = \dot{w}_0 (1 - x) , \quad \dot{v} = \dot{w}_0 \left(x - \frac{x^2}{2} \right) ;
\end{aligned} \qquad /4.8/
$$

in this case expression /4.4/ becomes:

$$\dot{\mu} = \frac{\frac{2}{3} \alpha \dot{w}_0 \left(3t \frac{A^2}{L^2} + 2t - 1 - \frac{5}{\alpha} \right)}{\alpha + 2} . \qquad /4.9/$$

Hence

$$\dot{\mu} > 0 \quad \text{for} \quad t > \frac{\alpha + 5}{3\frac{A}{H} + 2\alpha} \quad \text{shell is stable} , \qquad /4.10/$$

$$\dot{\mu} < 0 \quad \text{for} \quad t < \frac{\alpha + 5}{3\frac{A}{H} + 2\alpha} \quad \text{shell is unstable} . \qquad /4.11/$$

For the range of loads represented by the line BC or B'C' in Fig. 2 the limit load solution [5] gives:

$$
\begin{aligned}
&-1 < t < 0 , \quad p = 1 + \frac{2}{\alpha} , \\
&m_x = 1 - 2x^2 , \quad n_\theta = -1 , \quad n_x = t , \\
&\dot{w} = \dot{w}_0 (1 - x) , \quad \dot{v} = 0 .
\end{aligned} \qquad /4.12/
$$

The load intensity /4.4/ reduces to the form

$$\dot{\mu} = \frac{2\alpha \dot{w}_0 \left(3t \frac{A^2}{L^2} - 1 - \frac{3}{\alpha} \right)}{3(\alpha + 2)} . \qquad /4.13/$$

Since numerator is always negative and denominator is always positive we have

$$\dot{\mu} < 0 , \qquad /4.14/$$

hence the shell is unstable for any t from the range $(-1, 0)$.

For the range of loads represented by the line AB or A'B' in Fig. 2, limit load solution [5] gives:

$$t = -1$$
$$m_x = k(1-2x^2) \;, n_x = -1 \;, \; n_\theta = -p + \frac{2k}{\alpha} \qquad /4.15/$$
$$\dot{w} = 0 \;, \quad \dot{v} = -\dot{v}_o x$$

where $-1 \leqslant k \leqslant 1$.

The load intensity calculated from /4.4/ gives:

$$\dot{\mu} = -\dot{v}_o < 0 \qquad /4.16/$$

and therefore the shell is unstable.

Analogous calculations can be performed for negative values of the load p .

The stability, in the case considered, depends on the ratio of the surface tractions as well as on the geometry of the shell described by the ratio $\frac{A}{L}$ and the parameter α . The yield-point loading curves can be divided, by the lines $H-H'$ and $K-K'$, into two parts (Fig. 2). One of them /denoted by the signs $\oplus$ / corresponds to a stable and the other /denoted by the sign $\ominus$ / to an unstable limit state solutions. The results presented on Fig. 2 are calculated for $L/A = 1, 2, \infty$.

Let us now compare the results obtained above at the assumption of incompressibility, with those presented in the paper [6] calculated for the same example of the shell but under the assumption of constant shell thickness and the displacement field in the form /4.3/. Fig. 3 shows the comparison of results for $L/A = 1, 2, \infty$.

The results presented show that such fenomena as material incompressibility may play important role when stability of plastic shell, is considered. The differences are grater for thicker shells / α small/, whereas, the both solutions coincide for infinitely thin shells / $\alpha = \infty$ /. The lined zone in Fig. 4 indicates the

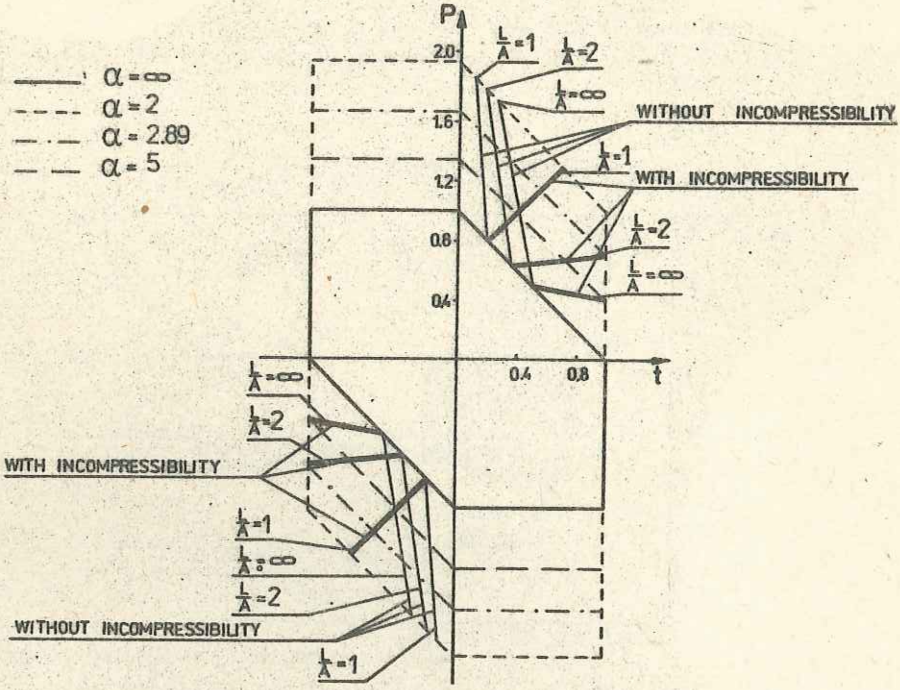

Fig. 3

ratio of the loads p_o/t for which the answer whether the shell at the yield point load is stable or unstable, changes if the incompressibility condition is taken into account.

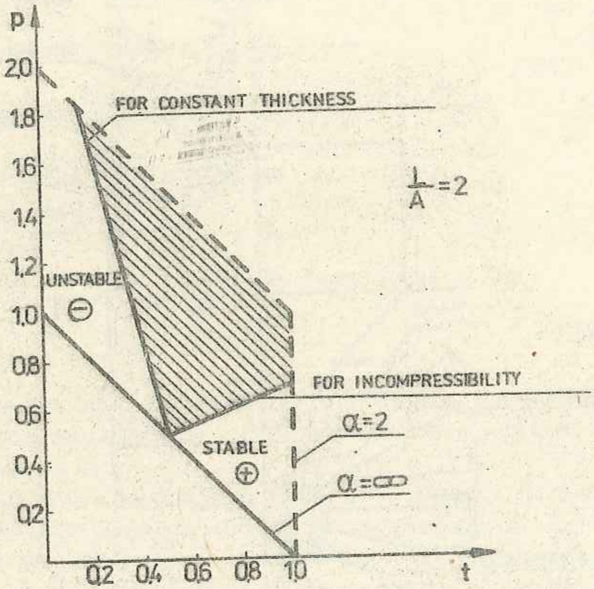

Fig. 4

We now proceed to investigate the stability problem for the same example of cylindrical shell as considered above but made of rigid perfectly plastic material in the Jaumann sense. Making use of /2.11/ in /3.4/ the expression defining the rate of load intensity for perfectly plastic material in the Jaumann sense may be written as

$$\dot{\mu} = \frac{-\int_V S^{KL} \dot{U}_{K|M} \dot{U}^M_{\ |L} \, dV}{\int_S \mathring{P}^K \dot{U}_K \, dS} . \qquad /4.17/$$

Next, analogously as before, for the displacement field /4.2/ and the dimensionless quantities /4.5/ - /4.7/, the equation /4.17/ can be rewritten in the form

$$\dot{\mu} = \frac{\int_S \left[-n_x \left(\dot{v}_{,x}^2 - \frac{A^2}{L^2} \dot{w}_{,x}^2 \right) - \frac{1}{2\alpha} m_x \left(\dot{w}_{,x} \dot{v}_{,xx} + \dot{w}_{,x}^2 - 2 \dot{v}_{,x} \dot{w}_{,xx} \right) - \right.}{\int_S (p\dot{w} + t\dot{v})_{|x=1} \, dS} \qquad /4.18/$$

$$\frac{\left. - n_\theta \dot{w}^2 + \frac{1}{\alpha} m_{x,x} \dot{w}_{,x} \dot{w} \right] dS}{} .$$

Substituting the limit load solution /4.8/ for $0 \leq t < 1$, into /4.18/ we obtain

$$\dot{\mu} = \frac{2 \dot{w}_0 \alpha \left(3t \frac{A^2}{L^2} - 2t + 1 - \frac{2}{\alpha} \right)}{3(\alpha + 2)} . \qquad /4.19/$$

Hence, the shell is stable, $\dot{\mu} > 0$

for $t < \dfrac{2-\alpha}{\alpha(3\frac{A^2}{L^2} - 2)}$ if $\dfrac{A^2}{L^2} < \dfrac{2}{3}$

and for any t if $\frac{A^2}{L^2} > \frac{2}{3}$.

Whereas the shell is unstable, $\dot{\mu} < 0$

for $t > \dfrac{2-\alpha}{\alpha(3\frac{A^2}{L^2} - 2)}$ if $\dfrac{A^2}{L^2} < \dfrac{2}{3}$.

Similarly, substitution of /4.12/ into /4.18/ leads for $-1 < t < 0$ to

$$\dot{\mu} = \frac{2 \dot{w}_0 \alpha \left(3t \frac{A^2}{L^2} + 1 + \frac{3}{2\alpha} \right)}{3(\alpha + 2)} \qquad /4.20/$$

hence, the shell is stable

$$\dot{\mu} > 0 \qquad \text{for } t > \frac{-3-2\alpha}{6\alpha\frac{A^2}{L^2}} , \qquad /4.21/$$

and the shall is unstable

$$\dot{\mu} < 0 \qquad \text{for } t < \frac{-3-2\alpha}{6\alpha\frac{A^2}{L^2}} . \qquad /4.22/$$

Finally, for $t=-1$, substitution of /4.15/ into /4.18/ furnish

$$\dot{\mu} = \dot{v}_0 > 0 \qquad /4.23/$$

what indicates that the shell is stable. Foregoing results, for the cylindrical shells made of incompressible, perfectly plastic material in the Jaumann sense are presented in Fig. 5 for $\alpha = 2 \div \infty$, $L/A = 1$ and in Fig. 6 for $\alpha = 2 \div \infty$, $L/A = 2$.

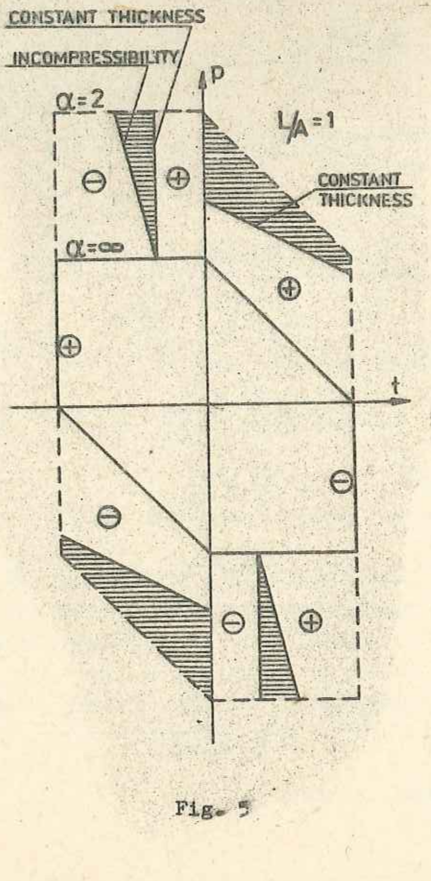

Fig. 5

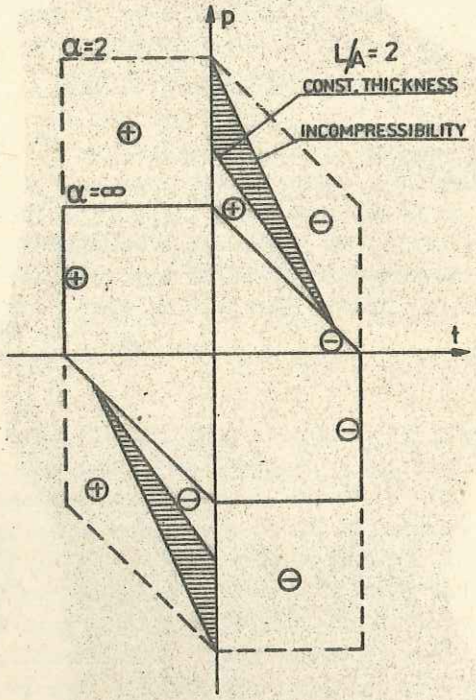

Fig. 6

For the same definition of perfectly plastic material (in the Jaumann sense) but for the displacement field /4.3/ /at the Kirchhoff-Love assumption of constant thickness/, the stability problem at the yield point load has the following solution:

For $0 \leq t < 1$:

$$\dot{u} = \frac{2\dot{w}_o \alpha \left(3t \frac{A^2}{L^2} - 2t + 1 - \frac{5}{\alpha}\right)}{3(\alpha + 2)} \, , \qquad /4.24/$$

for $-1 < t < 0$:

$$\dot{u} = \frac{2\dot{w}_0 \alpha (3t \frac{A^2}{L^2} + 1)}{3(\alpha + 2)}, \qquad /4.25/$$

for $t = -1$:

$$\dot{u} = \dot{v}_0 \quad . \qquad /4.26/$$

Figures 5 and 6 illustrates those results for $\alpha = 2 \div \infty$ and $\frac{L}{A} = 1, 2$.

Presented pictures illustrate two typical situations. For $L/A > 2$ the distribution of stable and unstable zones are similar to the case of $L/A = 2$. The lined zones marked in the Figs. 5 and 6 indicate the ratio of p/t for which the answer whether the shell is stable or unstable changes when the incompressibility condition is taken into account in considerations.

5. Conclusions

Presented results indicate that the influunce of material incompressibility on the post yield behaviour of cylindrical shells can be essential in the both situations, if perfectly plastic material in the Oldroyd sense or in the Jaummann sense is assumed. The different answer on the question of stability at the yield point load if incompressibility condition is taken into account, is obtained for the large zone in $t - p$ space when the shell is rather thick / α - small/, whereas the both solutions coincide for the infinitely long shell / $\alpha = \infty$ /. For perfectly plastic material in the Jaummann sense the post yield behaviour is more complex th n in the case of perfectly plastic material in the Oldroyd sense. The influence of incompressibility depends both on the ratio of loads t/p and on the shape of the shell given by parameters L/A and α .

There is no evident proof which definition of plastic material is more suitable for description of the behaviour of shells made of mild steel, however available experimental data [7] are in agreement with solutions obtained for the shells made of perfectly plastic material in the Oldroyd sense.

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Abstract

The paper deals with the problem of stability of rigid-perfectly plastic shells at the yield point load.

The material incompressibility, observed when plastic deformations take place, is taken into account and the results are compared with the known solution obtained when the thickness of the shell is assumed to be constant.

The influence of different definitions of perfectly plastic material on the post yield behaviour of the shells is also considered.

The question is illustrated by the examples of cylindrical shells subject to axial end forces and uniformly distributed lateral dead load.